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## BOOK OF ABSTRACTS

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## Topics on the analysis of recurrent events

Ana Maria Abreu, Ivo Sousa-Ferreira, Cristina Rocha

Over the last decades, there have been several developments in the analysis of recurrent events. Many of them arose as extensions of Cox's regression model (e.g., Prentice, Williams and Peterson's model or Andersen and Gill's model), i.e., as a semiparametric approach. However, a parametric approach or a compromise between the previous ones is possible. In order to capture how the occurrence risk varies over time, considering a flexible hazard function is usually a good option. This can be achieved, for instance, by using an adequate lifetime distribution or restricted cubic splines.

The aim of this work is to present some topics on the analysis of recurrent events, discussing the formulation of new survival models that differ in terms of the distributional assumptions. In particular, the use of a probability distribution recently proposed by Sousa-Ferreira et al. [1, 2], or restricted cubic splines, is explored.

### References

- [1] Sousa-Ferreira, I., Abreu, A.M. and Rocha, C. (In press). The extended Chen-Poisson lifetime distribution. *REVSTAT*, Accepted for publication, november, 2021.
- [2] Sousa-Ferreira, I., Rocha, C. and Abreu, A.M. (2022). The extended Chen-Poisson marginal rate model for recurrent gap time data. In R. Bispo, L. Henriques-Rodrigues R., Alpizar-Jara and M. de Carvalho (eds.), *Recent Developments in Statistics and Data Science*, pp. 337-351. Springer Proceedings in Mathematics & Statistics, vol. 398. Springer, Cham.

## Invariant curves for piecewise isometries

Ana Rodrigues

In this talk, I will review some recent results on the study of the dynamics of Piecewise Isometries (PWIs). I will give an overview of the notion of an embedding of an interval exchange transformation into a PWI, discuss the renormalization of a particular piecewise isometry and finally show that invariant curves exist for such transformations. (This is joint work with Arek Goetz, Peter Ashwin, Pedro Peres and Noah Cockram).

### References

- [1] Embeddings of interval exchange transformations into planar piecewise isometries (with P. Ashwin, A. Goetz, P. Peres). *Ergod. Th. Dynam. Sys.* **40**, Issue 5, (2020), 1153 – 1179.
- [2] P. Peres and A. Rodrigues. Existence of non-trivial embeddings of interval exchange transformations into Piecewise Isometries. Submitted.
- [3] P. Peres and A. Rodrigues. Dynamics of planar piecewise isometries: Recent advances. *Boletim da Sociedade Portuguesa de Matemática* – special volume (by invitation) for celebrating the 100th anniversary of the Portuguese Mathematical Society.
- [4] P. Peres and A. Rodrigues. On the dynamics of translated cone exchange transformations (with P. Peres). To be submitted soon.
- [5] P. Ashwin, N. Cockram, A. Rodrigues. Construction of invariant curves for some planar piecewise isometries. In preparation.

## And now worlds may vary!

Bruno Dinis (presenter), Diana Costa

In this talk I introduce an extension of Hybrid Logic, dubbed VHL, in which quantifier domains (over world and domain variables) may vary from world to world. Relying on a novel notion of prenex form, it is possible to introduce a new form of Skolemization, which is a non-trivial refinement of the usual process. These concepts are strongly dependent on the @ operator, and are therefore referred to as @-prenex normal form, and @-Skolemization. I give a very broad overview of the steps necessary to prove a Herbrand-like theorem for VHL, stating the equivalence between the satisfiability of a set of formulas and the propositional satisfiability of the ground instances of an @-Skolemized version of these formulas.

### References

- [1] Patrick Blackburn. Representation, reasoning, and relational structures: A hybrid logic manifesto. *Logic Journal of the IGPL*, 8(3):339–365, 2000.
- [2] Patrick Blackburn, Manuel Martins, María Manzano, and Antonia Huertas. Rigid first-order hybrid logic. In *Logic, language, information, and computation*, volume 11541 of *Lecture Notes in Comput. Sci.*, pages 53–69. Springer, Berlin, 2019.
- [3] Patrick Blackburn, Manuel Martins, María Manzano, and Antonia Huertas. Exorcising the phantom zone. *Inform. and Comput.*, 287:Paper No. 104754, 21, 2022.
- [4] Diana Costa, Manuel A. Martins, and João Marcos. On Herbrand’s theorem for hybrid logic. *J. Appl. Logics*, 6(2):209–228, 2019.
- [5] Diana Costa, Bruno Dinis. On Satisfiability for Hybrid Logic with Varying Domains (submitted)

## General environmental stochasticity models for harvested populations: impact of Allee effects

Carlos A. Braumann (presenter), Clara Carlos, Nuno M. Brites

We study general autonomous stochastic differential equation (SDE) models of harvested populations with Allee effects living in a randomly varying environment of the form  $\frac{dX(t)}{X(t)} = f(X(t))dt + \sigma(X(t))dW(t) - qE(X(t))dt$ , where  $X(t)$  is the population size at time  $t$ ,  $f$  is the arithmetic average *per capita* growth rate,  $\sigma > 0$  is the intensity of the effect of environmental fluctuations on the growth rate,  $W(t)$  is the standard Wiener process,  $E \geq 0$  is the harvesting effort, and  $q > 0$  is the catchability. Besides mild regularity assumptions, the functions  $f$ ,  $E$  and  $\sigma$  are quite general, obviously with  $f$  satisfying biologically-driven properties of populations under Allee effects. For such general models, we obtain conditions for extinction and conditions for the existence of a stochastic equilibrium with a stationary density, as well as the expression for such a density.

We show that, if the *per capita* net growth rate (difference between the geometric average natural growth rate and the harvesting mortality rate) is positive when population size is very small, there is a stochastic equilibrium with a stationary density. If, however, that rate is negative (overharvesting), the population becomes extinct.

These results extend previous results for models of this type obtained in [1] (for the case of constant noise intensity  $\sigma(X) \equiv \sigma$ ) and in [2] (for general  $\sigma(X)$ ) for populations without Allee effects, in which case  $f$  satisfies biologically-driven different properties, those that characterize populations with no Allee effects. The comparison between models with and without Allee effects allows us to assess the impact of such effects, as was done in [3] for particular cases.

Our results also extend the results obtained in [4] for populations with Allee effects but with constant noise intensity ( $\sigma(X) \equiv \sigma$ ) and without harvesting ( $E(X) \equiv 0$ ).

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## References

- [1] Braumann, C.A. (1999). Variable effort fishing models in random environments. *Mathematical Biosciences* 156: 1–19. [https://doi.org/10.1016/S0025-5564\(98\)10058-5](https://doi.org/10.1016/S0025-5564(98)10058-5)
- [2] Braumann, C.A. (2002). Variable effort harvesting models in random environments: generalization to density dependent noise intensities. *Mathematical Biosciences* 177 & 178: 229–245. [https://doi.org/10.1016/S0025-5564\(01\)00110-9](https://doi.org/10.1016/S0025-5564(01)00110-9)
- [3] Brites, N.M., Braumann, C.A. (2022). Profit optimization of stochastically fluctuating populations under harvesting: the effects of Allee effects. *Optimization* 71(11): 3277–3293. <https://doi.org/10.1080/02331934.2022.2031191>
- [4] Carlos, C., Braumann, C.A. (2017). General population growth models with Allee effects in a random environment. *Ecological Complexity* 30: 26–33. <http://dx.doi.org/10.1016/j.ecocom.2016.09.003>

## Impact of Allee effects on constant effort harvesting of populations in random environments: logistic type models

Clara Carlos (presenter), Nuno M. Brites, Carlos A. Braumann

Conditions for the existence of a stochastic equilibrium for general autonomous stochastic differential equation (SDE) models of population growth with harvesting in a randomly varying environment have been presented in [1]. However, Allee effects were not considered. They were considered in [3] but only for populations not being harvested. In our other communication on this Annual Meeting, we extend such results to general autonomous SDE models of populations with Allee effects under harvesting of the form  $\frac{dX(t)}{X(t)} = f(X(t))dt + \sigma(X(t))dW(t) - qE(X(t))dt$ , where  $X(t)$  is the population size at time  $t$ ,  $f$  is the arithmetic average *per capita* growth rate,  $\sigma > 0$  is the intensity of the effect of environmental fluctuations on the growth rate,  $W(t)$  is the standard Wiener process,  $E \geq 0$  is the harvesting effort, and  $q > 0$  is the catchability. Besides mild regularity assumptions, the functions  $f$ ,  $E$ , and  $\sigma$  are quite general, obviously with  $f$  satisfying biologically-driven properties of populations under Allee effects.

We now apply those results to specific models commonly used in applications, namely the logistic model (without Allee effects)  $f(X) = r(1 - \frac{X}{K})$  ( $r, K > 0$ ) and the logistic-like Allee effects model  $f(X) = r(1 - \frac{X}{K})\left(\frac{X-A}{K-A}\right)$  ( $r, K > 0; A \in (-K, K)$ ); reparametrized as in [3] to make it comparable with the logistic model) for the case of constant effort harvesting  $E(X) \equiv E$  and constant noise intensity  $\sigma(x) \equiv \sigma$ . Conditions for the existence of a stationary density and the expressions for such densities and for the expected sustainable profits obtained in [2] are presented, as are the expressions for the optimal sustainable yields.

This allows us to assess the impact of Allee effects for different values of the Allee parameter  $A$  by comparing the two models and their optimal sustainable profits (reporting results in [2]), as well as their optimal sustainable yields, for the Pacific halibut data in [4].

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## References

- [1] Braumann, C.A. (2002). Variable effort harvesting models in random environments: generalization to density dependent noise intensities. *Mathematical Biosciences* 177 & 178: 229–245. [https://doi.org/10.1016/S0025-5564\(01\)00110-9](https://doi.org/10.1016/S0025-5564(01)00110-9)
- [2] Brites, N.M., Braumann, C.A. (2022). Profit optimization of stochastically fluctuating populations under harvesting: the effects of Allee effects. *Optimization* 71(11): 3277–3293. <https://doi.org/10.1080/02331934.2022.2031191>
- [3] Carlos, C., Braumann, C.A. (2017). General population growth models with Allee effects in a random environment. *Ecological Complexity* 30: 26–33. <http://dx.doi.org/10.1016/j.ecocom.2016.09.003>
- [4] Hanson, F.B., Ryan, D. (1998). Optimal harvesting with both population and price dynamics. *Mathematical Biosciences* 148(2): 129–146. [https://doi.org/10.1016/S0025-5564\(97\)10011-6](https://doi.org/10.1016/S0025-5564(97)10011-6)

## Classic and deep learning techniques for temporal clustering and mathematical modeling

Dulce Gomes, Cristiana J. Silva, Patrícia A. Filipe

Classification Data Analysis is an extremely important area of research and application, having gained in recent years more and more importance due to the gigantic capacity of collecting, storing and processing data. Analytical methods have been widely applied and has proved to be extremely useful tool for dealing with a large volume of information.

For these reasons, we have witnessed the development of new methodologies for classifying and grouping data. The classification of data according to their geographic and temporal similarities and dissimilarities has not only been an exception, but has also had enormous importance due to the exponential growth of georeferenced and temporal databases in recent years.

As time series are unlabeled datasets, the challenge of classifying them and validating their effectiveness becomes more complex. Similarity metrics such as Euclidean distance or Manhattan distance are quite common in clustering processes, but only appropriate for data that do not present correlation. However, when working with time series, it is essential to take into account the correlation in the similarity measures.

There are several different approaches to time series clustering, according to the similarity metrics between time series and to the clustering methods. Due to the specific nature of time series data, a Dynamic Time Warping distance (DTW) - a dissimilarity measure that is based on a temporal proximity - has been studied and applied in the context of COVID-19 [1]. DTW has the main advantage of group time series according to their patterns or shapes, even if these patterns are not perfectly lag-synchronized. Furthermore, DTW is an accurate and computationally efficient distance measure “providing both a distance measure that is insensitive to local compression and stretches and the warping which optimally deforms one of the two input series onto the other.”[2].

We intend to study and compare the DTW with other measures of similarity/dissimilarity, considering different dependency structures between observations - for example, stationary series, non-stationary series (with trend and/or seasonality), with structural change -, exploring the difference of work with the raw time series or with certain transformations.

In unsupervised problems, such as time series clustering, there are no labels to establish performance metrics. Therefore, simulation studies will be implemented for this evaluation and comparative validation.

The hierarchical clustering has been the algorithm used to separate groups according to data similarities. This has the advantage to produce a hierarchical series of nested clusters that can be represented graphically by dendrograms. Through the branches of the dendrogram it is possible to identify not only the similarities between the clusters, but also to know the conformation of the

clusters. However, this algorithm has, like so many others, advantages and disadvantages, so we will study and compare the performance of other algorithms.

Based on the clustering analysis, we can study the groups with data similarities and use compartmental models to describe their variation in time, with respect to some characteristic, like, for example, health status. We will study compartmental models for the transmission of infectious diseases, defined by ordinary differential equations, such as SIR (Susceptible, Infected, Recovered), SIRD (Susceptible, Infected, Recovered, Dead), among others, and apply them to our data. We will investigate methods for parameters estimation and forecasting, suitable for this type of mathematical models, and compare the results with and without the use of the clustering analysis. After calibrating suitable compartmental models to the groups with data similarities, we can model the effect of migrations between groups by constructing complex networks [4]. All these methods become a difficult task if we manage an important amount of data. With the extraordinary advances in Artificial Intelligence techniques, deep learning models have proved to be highly effective in predicting the future evolution of infectious diseases based on huge sets of historical data.

We will also intend to explore the advanced analytical methods, namely the deep learning-based methods for clustering time series analysis - Deep Time-Series Clustering (DTSC) [3] – and compare the performance with the “traditional algorithms”.

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## References

- [1] Gomes, D; Zamboni Berra, T.; Ramos, A; Arcêncio, R.A. Hierarchical clustering algorithms in classification the spread of COVID-19 cases and deaths in European countries. *XXV Conference of the Portuguese Statistical Society, October 13–16, 2021*
- [2] Giorgino, T. Computing and Visualizing Dynamic Time Warping Alignments in R: The dtw Package, *J. Stat. Softw.*, 2009, 31(7).
- [3] Alqahtani, A.; Ali, M.; Xie, X.; Jones, M.W. Deep Time-Series Clustering: A Review. *Electronics* 2021, 10, 3001.
- [4] Cantin, G.; Silva, C. J. Influence of the topology on the dynamics of a complex network of HIV/AIDS epidemic models. *AIMS Mathematics*, 2019, 4(4): 1145-1169.

## Heteroclinic solutions for strongly nonlinear differential equations

Feliz Minhós

In this talk, we consider the second order discontinuous differential equation in the real line,

$$(a(t, u) \varphi(u))' = f(t, u, u'), \quad a.e.t \in \mathbb{R},$$

with  $\varphi$  an increasing homeomorphism such that  $\varphi(0) = 0$  and  $\varphi(\mathbb{R}) = \mathbb{R}$ ,  $a \in C(\mathbb{R}^2, \mathbb{R})$  with  $a(t, x) > 0$ , for  $(t, x) \in \mathbb{R}^2$ ,  $f : \mathbb{R}^3 \rightarrow \mathbb{R}$  a  $L^1$ -Carathéodory function, and the asymptotic conditions

$$u(-\infty) = \nu^-, u(+\infty) = \nu^+,$$

where  $\nu^-, \nu^+ \in \mathbb{R}$ , such that  $\nu^- < \nu^+$ .

The existence and localization of heteroclinic connections is obtained assuming a Nagumo-type condition on the real line and without asymptotic conditions on the nonlinearities  $\varphi$  and  $f$ .

To the best of our knowledge, this result is even new when  $\varphi(y) = y$ , that is for equation

$$(a(t, u(t)) u'(t))' = f(t, u(t), u'(t)), \quad a.e.t \in \mathbb{R}.$$

Moreover, these results can be applied to classical and singular  $\varphi$ -Laplacian equations and to the mean curvature operator.

**Keywords:**  $\varphi$ -Laplacian operator; mean curvature operator; heteroclinic solutions; problems in the real line; lower and upper solutions; Nagumo condition on the real line; fixed point theory.

**Mathematics Subject Classification:** 34C37; 34B40; 34B15; 47H10

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## A Lagrange multiplier for imprecise objective functions and precise constraints

Imme van den Berg, with Nam Van Tran (HCMUTE, Vietnam)

A Lagrange multiplier theorem is derived for a class of optimization problems where the objective function is set-valued and the constraint is given by a real-valued function [3]. This extends sensitivity analysis [4] to the objective function, typically modelling the situation where our goal is imprecise due to all kinds of uncertainties, but our budget is well-determined. We define a sort of macroscopic derivative, ignoring small-scale uncertainties and quivering [1]. The derivatives satisfy the usual laws for the algebraic operations and the chain rule with inclusions. These laws permit to control the error terms when deriving versions of the Fermat Lemma and the Implicit Function Theorem [2], which are the common tools used in the proof of the multiplier theorem.

### References

- [1] Almeida, R., Neves, V.: A strong form of almost differentiability. *J. Math. Sci.* 161(6), 894–908 (2009)
- [2] Dontchev, A.L., Rockafellar, R.T.: *Implicit Functions and Solution mappings*. Springer, Berlin (2008)
- [3] Tran, N.V., van den Berg, I. (2022). On Non-linear Optimization with a Perturbed Objective Function. In: Tchemisova, T.V., Torres, D.F.M., Plakhov, A.Y. (eds) *Dynamic Control and Optimization. DCO 2021*. Springer Proceedings in Mathematics & Statistics, vol 407. Springer, Cham. [https://doi.org/10.1007/978-3-031-17558-9\\_3](https://doi.org/10.1007/978-3-031-17558-9_3)
- [4] Saltelli, A., Ratto, M., Andres, T., Campolongo, F., Cariboni, J., Gatelli, D., Saisana, M., Tarantola, S.: *Global Sensitivity Analysis: The Primer*. Wiley, New York (2008)

# Shintani Descent and Supercharacter Theory

J. Dias

The problem of describing the representation theory of the unitriangular group (the group of upper triangular matrices with diagonal entries set to 1) led to the development of supercharacter theory, first by C. André for the unitriangular group over a finite field and later, for a general finite group, by I.M. Isaacs and P. Diaconis.

The Shintani Descent, first introduced by T. Shintani, was used to describe the relation between  $F$ -stable irreducible characters of an algebraic group  $G(\mathbb{F}_{q^n})$  and characters of  $G(\mathbb{F}_q)$ . Whereas, in the case where  $G$  is abelian, we get a bijection between  $F$ -irreducibles and irreducibles, but for general cases, we do not have such correspondence.

In this talk, I will give an introduction to the usual supercharacter theory defined on a unitriangular group over a finite field and, using the Shintani Descent, describe a relation between the supercharacter theory of the unitriangular group of a finite field and over a finite extension of such a field. Lastly, some connections with the representation theory of the unitriangular group over an infinite field will be explored.

## References

- [1] Deshpande, Tanmay. "Shintani descent for algebraic groups and almost characters of unipotent groups." *Compositio Mathematica* 152.8 (2016): 1697-1724.
- [2] Shoji, Toshiaki. "Shintani descent for algebraic groups over a finite field, I." *Journal of Algebra* 145.2 (1992): 468-524.
- [3] Diaconis, Persi, and I. Isaacs. "Supercharacters and superclasses for algebra groups." *Transactions of the American Mathematical Society* 360.5 (2008): 2359-2392.
- [4] André, Carlos, and Dias, João, Shintani Descent for standard supercharacters of algebra groups, *submitted*



## The E-SEASON Project

Joaquim M. C. Correia

E-SEASON is the acronym for Europe-South East Asia Science Oriented Network which is an informal network enrolling a number of European and Asian universities and having as kernel of all of its activities: mathematics. We want to present the E-SEASON, the academic cooperation developed inside as well as the ongoing projects. In particular we will pay attention to its research goals.

## Recent contributions in risk assessment

Lígia Henriques-Rodrigues

In the field of statistical extreme value theory, risk is generally expressed either by the value at risk at a level  $q$  ( $\text{VaR}_q$ ), the size of the loss occurred with a fixed probability,  $q$ , the upper  $(1 - q)$ -quantile of the loss function, or by the conditional tail expectation (CTE), defined as  $\text{CTE}_q = E(X|X > \text{VaR}_q)$ ,  $q \in (0, 1)$ . The value of  $q$  is often smaller than  $1/n$ , where  $n$  denotes the size of the available sample,  $\mathbf{X}_n = (X_1, \dots, X_n)$ . We consider heavy-tailed models, i.e. Pareto-type underlying cumulative distribution functions, with a positive extreme value index (EVI), quite common in many areas of application. For these Pareto-type models, the classical EVI-estimators are the Hill estimators introduced in [1], the average of the  $k$  log-excesses over a threshold  $X_{n-k:n}$ . The Hill estimator is crucial for the semi-parametric estimation of the CTE. The class of CTE-estimators considered is the one proposed in [2] (see also [3]). In this presentation, we showcase advancements in the performance of the aforementioned CTE-estimators. These improvements are achieved by utilizing reliable EVI-estimators based on generalized means, such as the ones introduced in [4, 5, 6] (see also [7]) and incorporating reduced-bias EVI-estimators introduced in [8, 9, 10].

## References

- [1] Hill, B.M. (1975). A simple general approach to inference about the tail of a distribution. *Ann. Statist.* **3**, pp. 1163–1174.
- [2] Necir, A., Rassoul, A. and Zitikis, R. (2010). Estimating the conditional tail expectation in the case of heavy-tailed losses. *Journal of Probability and Statistics*, **2010**, Article ID 596839, 17 pages.
- [3] Laidi, M., Rassoul, A. and Old Rouis, H. (2020). *Improved Estimator of the Conditional Tail Expectation in the case of heavy-tailed losses*. *Statistics, Optimization & Information Computing*, **8**:1, pp. 98–109.
- [4] Caeiro, F., Gomes, M.I., Beirlant, J. and de Wet, T. (2016). Mean-of-order  $p$  reduced-bias extreme value index estimation under a third-order framework. *Extremes* **19**:4, pp. 561–589.
- [5] Penalva, H., Gomes, M.I., Caeiro, F. and Neves, M.M. (2020). Lehmer’s mean-of-order- $p$  extreme value index estimation: a simulation study and applications. *J. Applied Statistics* **47**:13-15 (*Advances in Computational Data Analysis*), pp. 2825–2845.
- [6] Gomes, M.I. and Martins, M.J. (2001). Generalizations of the Hill estimator – asymptotic versus finite sample behaviour, *Journal of Statistical Planning and Inference*, **93**, pp. 161–180.

- [7] Gomes, M.I., Figueiredo, F. and Henriques-Rodrigues, L. (2023). Reliable alternative ways to manage the Risk of Extreme Events (to appear in a Springer book).
- [8] Caeiro, F., Gomes, M.I. and Pestana, D. (2005). Direct reduction of bias of the classical Hill estimator. *Revstat—Statist. J.*, **3**:2, pp. 111–136.
- [9] Gomes, M.I., Brilhante, M.F. and Pestana, D. (2016). New reduced-bias estimators of a positive extreme value index. *Comm. in Statistics—Simulation and Computation*, **45**, pp. 1–30.
- [10] Gomes, M.I., Penalva, H., Caeiro, F. and Neves, M.M. (2016). Non-reduced versus reduced-bias estimators of the extreme value index—efficiency and robustness. In A. Colubi *et al.* (eds.), COMPSTAT 2016—22nd International Conference on Computational Statistics, ISI/IASC, pp. 279–290.

## $\mathcal{A}$ -policonvexidade\*

Luís Bandeira

Embora a condição de  $\mathcal{A}$ -quasiconvexidade tenha sido totalmente explorada desde a sua introdução, nenhum exemplo explícito de princípios variacionais associados foi considerado, exceto no caso clássico do gradiente. O nosso objetivo é propor tal família de problemas na situação  $\text{div} - \text{curl}$ , e explorar a condição de  $\mathcal{A}$ -policonvexidade correspondente como a principal hipótese estrutural para garantir a semicontinuidade inferior e, conseqüentemente, resultados de existência de solução.

### References

- [1] L. Bandeira, P. Pedregal,  $\mathcal{A}$ -variational principles, *Milan Journal of Mathematics* (2023).

\*Trabalho conjunto com Pablo Pedregal, durante a visita de L. Bandeira ao Omeva Research Group na Universidad de Castilla-La Mancha, Espanha, ano académico de 2022/2023.

## Belief change: basic concepts and some recent contributions

Maurício Reis, Marco Garapa

The one that is nowadays considered the standard model of belief change is the AGM model, proposed by Alchourrón, Gärdenfors and Makinson [1]. Since it was first presented, many researchers have proposed numerous extensions and generalizations of this framework (for an overview see [2, 3]).

In this talk we will present the AGM model and the main motivations underlying its proposal. Furthermore we will present some extensions and generalizations of that model that we have recently proposed in [4, 5, 6].

### References

- [1] Carlos Alchourrón, Peter Gärdenfors, and David Makinson. On the logic of theory change: Partial meet contraction and revision functions. *Journal of Symbolic Logic*, 50:510–530, 1985.
- [2] Eduardo Fermé and Sven Ove Hansson. AGM 25 years: Twenty-five years of research in belief change. *Journal of Philosophical Logic*, 40:295–331, 2011.
- [3] Eduardo Fermé and Sven Ove Hansson. *Belief Change: Introduction and Overview*. Springer Briefs in Computer Science Series. Springer, 2018.
- [4] Marco Garapa. Two level credibility-limited revisions. *The Review of Symbolic Logic*, pages 1 – 21, 2020.
- [5] Marco Garapa, Eduardo Fermé, and Maurício D.L. Reis. System of spheres-based two level credibility-limited revisions. In *Proceedings of the 19th Conference on Theoretical Aspects of Rationality and Knowledge, TARK 2023, Oxford, UK, June 28-30, 2023*. EPTCS (Electronic Proceedings in Theoretical Computer Science), 2023.
- [6] Marco Garapa and Maurício D.L. Reis. Generalized partial meet and kernel contractions. *The Review of Symbolic Logic*, page 129, 2022.

## Pendent steady rivulets: from lubrication to bifurcation

Michael Grinfeld

We consider the shape of the free surface of steady pendent rivulets beneath a planar substrate. We formulate the governing equations in terms of two closely related dynamical systems and analyse the bifurcation structure of the problem. Our results explain why lubrication theory is unable to describe the structure of the solutions set for pendent rivulets, although it is successful in the related problem of sessile rivulets, and show how to construct rivulets with overhang.

This is joint work with David Pritchard.

## An approach to social intervention and health problems

Nuno Franco

We present works that deal with two different kind of problems:

Participation in projects “Virtual Reality in the Social Inclusion of the Elderly” with several IPSS. This project intends to improve the physical, social and emotional conditions of the elderly. This improvement will be achieved through the implementation of a program, adapted to each reality and each individual (in depression or anxiety) through technology based on Virtual Reality, with experiences that promote the well-being and quality of life of the elderly, providing a positive social integration and promoting active and happy aging. A practical solution to decide how to feed a new born child with specific health problems.

The first topic is already finished and the second one is ongoing work.

## Stochastic differential equations mixed models to describe individual growth

Patrícia A. Filipe, Gonçalo Jacinto, Nelson T. Jamba,  
Carlos A. Braumann

Stochastic differential equations (SDE) models are adequate to describe the dynamic of individual growth in a randomly fluctuating environment. We have been studying estimation, prediction, and optimization issues for a class of SDE to model the evolution of cattle weight which, by an appropriate transformation of the weight, results in a variant of the Ornstein-Uhlenbeck model [1, 2, 3]. The model parameters are the average transformed weight at maturity  $\alpha$ , a growth parameter  $\beta$ , and the intensity of the effect of environmental fluctuations  $\sigma$ .

To incorporate individual characteristics of the animals, we have considered that the model parameters may vary randomly from animal to animal, which results in SDE mixed models. We have considered SDE mixed models where  $\alpha$  and/or  $\beta$  are i.i.d. Gaussian random variables. For these type of models, the model parameters are usually estimated by the maximum likelihood estimation method. However, in mixed models, this often becomes a difficult, sometimes impossible, task when the likelihood function involves integrals not solvable in closed-form, requiring the use of approximation methods. We propose a new approximation method, the delta method, which is easy to apply, and compare its performance with the Laplace method [4, 5]. This new method allows the estimation of the parameters even when the requirement of most known packages (for instance, [6]) fails. Namely, they require having the same time vector of observations for all trajectories. This is not the case in our application, since in our real data the animals are not weighted at the same ages and for this reason the existing packages could not be used.

We have also developed SDE mixed models that can incorporate specific individual characteristics of each animal. In particular, we propose SDE mixed models that incorporate each animal's genetic characteristics, by allowing  $\alpha$  to be a function of genetic values of the animal. The main goal is to extend the SDE mixed model to the more realistic case where an individual genetic value becomes an important component in the estimated growth curve. This will improve prediction of animal's future weights and help farmers optimize the profit obtained by raising and selling an animal.

### Acknowledgements

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## References

- [1] P. A. Filipe, C. A. Braumann, N. M. Brites, and C. J. Roquete. Modelling animal growth in random environments: an application using nonparametric estimation. *Biometrical journal*, **52**(5), 653-666, 2010.
- [2] G. Jacinto, P. A. Filipe, and C. A. Braumann. Profit optimization of cattle growth with variable prices. *Methodology and Computing in Applied Probability*, 24, 1917–1952, 2022.
- [3] G. Jacinto, P. A. Filipe, and C. A. Braumann (2022). Weighted maximum likelihood estimation for individual growth models. *Optimization*, **71**(11), 3295-3311, 2022.
- [4] N. T. Jamba, G. Jacinto, P. A. Filipe, and C. A. Braumann. Likelihood Function through the Delta Approximation in Mixed SDE Models. *Mathematics* **10**(3), 385, 2022.
- [5] N. T. Jamba, G. Jacinto, P. A. Filipe, and C. A. Braumann. Estimation for stochastic differential equations mixed models using approximation methods. *Submitted*
- [6] M. Delattre and C. Dion. MsdeParEst. Parametric Estimation in Mixed-Effects Stochastic Differential Equations. *R package version 1.7 1*.

## Jordan type of Artinian Gorenstein algebras and related invariants

Pedro Macias Marques

We study local Artinian Gorenstein (AG) algebras and consider the set of Jordan types of elements of the maximal ideal, i.e. the partition giving the Jordan blocks of the respective multiplication map.

In a joint work with Tony Iarrobino, we construct examples of families  $\text{Gor}(T)$  of local AG algebras with given Hilbert function  $T$ , and use obstructions that the symmetric decomposition of the associated graded algebra of an AG algebra  $A$  imposes on the Jordan type of  $A$  to study their irreducible components. Together with Johanna Steinmeyer, we explore possible generalisations of Jordan type and try to understand how they might apply to these topological questions.

## Spin(7)-structures on tangent bundles

Rui Albuquerque

The construction of a Spin(7)-structure on the tangent bundle of a 4-manifold follows quite as easily as that of gwistor space, ie. the  $G_2$ -structure on the tangent sphere bundle of a given oriented Riemannian 4-manifold. A PhD Dissertation of the University of Köln also constructs a natural structure on  $TM \setminus 0$  which arises as a weighted metric on  $\mathbb{R}$  with gwistor space. We compare the two notions and investigate the *problem* at the origin (the zero section).

### References

- [1] Stock, Sebastian Evolution of geometries with torsion. (English) Zbl 1296.53011 München: Dr. Hut; Köln: Univ. Köln, Mathematisch-Naturwissenschaftliche Fakultät (Diss.) (ISBN 978-3-86853-794-9/pbk). iii, 143 p. (2011).
- [2] Albuquerque, R. On the  $G_2$  bundle of a Riemannian 4-manifold. (English) Zbl 1190.53024 J. Geom. Phys. 60, No. 6-8, 924-939 (2010).

# First order non-linear coupled systems – existence and localization of periodic solutions using a method of upper and lower solutions

Sara Perestrelo, Feliz Minhós

We present some existence and localization results for periodic solutions of first-order coupled nonlinear systems of two equations, with and without impulses, without requiring periodicity for the nonlinearities. The arguments are based on Schauder's Fixed Point Theorem [1] together with the upper and lower solution method, where the upper and lower solutions are not necessarily well-ordered. In addition, for the impulsive analysis, results on equi-regulated functions [2, 3] are required. We apply our methodology to a Wilson-Cowan system of two strongly coupled neurons [4] to illustrate one of the main results.

## Acknowledgements

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## References

- [1] E. Zeidler, P.R. Wadsack, *Nonlinear Functional Analysis and Its Applications: Fixed-point Theorems/Transl. by Peter R. Wadsack*, Springer-Verlag, 1993.
- [2] D. Fraňková, Regulated functions, *Mathematica Bohemica*, 116(1):20–59, 1991. 10.21136/MB.1991.126195.
- [3] D. Fraňková, Regulated functions with values in banach space, *Mathematica Bohemica*, 144(4):437–456, 2019. 10.21136/MB.2019.0124-19.
- [4] T. Ueta, G. Chen, On synchronization and control of coupled wilson-cowan neural oscillators, *International Journal of Bifurcation and Chaos*, 13(01):163–175, 2003. <https://doi.org/10.1142/S0218127403006406>.



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**09:00-09:10**      **Opening of the Meeting**

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**Session 1**    **Chairman: José Luís Silva**

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09:10-09:30      General Environmental Stochasticity Models for Harvested Populations: Impact of Allee Effects  
Carlos A. Braumann (presenter), Clara Carlos, Nuno M. Brites

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09:30-09:50      Recent contributions in risk assessment  
Lígia Henriques-Rodrigues **ONLINE**

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09:50-10:10      Topics on the analysis of recurrent events  
Ana Maria Abreu, Ivo Sousa-Ferreira, Cristina Rocha **ONLINE**

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10:10-10:30      Impact of Allee effects on constant effort harvesting of populations in random environments.  
Logistic type models.  
Clara Carlos (presenter), Nuno M. Brites, Carlos A. Braumann

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10:30-10:50      Classic and Deep Learning Techniques for Temporal Clustering and Mathematical Modeling  
Dulce Gomes, Cristiana J. Silva, Patrícia A. Filipe

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**10:50-11:20**      **Coffee break**

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**Session 2**    **Chairman: Feliz Minhós**

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11:20-11:40      Pendent steady rivulets: from lubrication to bifurcation  
Michael Grinfeld

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11:40-12:00      Belief Change: Basic Concepts and Some Recent Contributions  
Maurício Reis and Marco Garapa

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12:00-12:20      A Lagrange multiplier for imprecise objective functions and precise constraints  
Imme van den Berg, with Nam Van Tran (HCMUTE, Vietnam)

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12:20-12:40      Stochastic differential equations mixed models to describe individual growth  
Patrícia A. Filipe, Gonçalo Jacinto, Nelson T. Jamba, Carlos A. Braumann

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**12:40-14:10 Lunch**

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**Session 3 Chairman: Carlos A. Braumann**

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14:10-14:30 Heteroclinic Solutions for Strongly Nonlinear Differential Equations  
Feliz Minhós14:30-14:50  $\mathcal{A}$ -policonvexidade  
Luís Bandeira14:50-15:10 First order non-linear coupled systems –Existence and localization of periodic solutions using a method of upper and lower solutions  
Sara Perestrelo15:10-15:30 Necessary and sufficient conditions for applicability of the pointwise constrained Liapunov convexity theorem in the 2x2 case  
Clara Carlota **ONLINE**15:30-15:50 And now worlds may vary!  
Bruno Dinis15:50-16:10 An approach to social intervention and health problems  
Nuno Franco**16:10-16:40 Coffee break**

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**Session 4 Chairman: Carlos Ramos**

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16:40-17:00 Invariant curves for Piecewise Isometries  
Ana Rodrigues17:00-17:20 Spin(7)-structures on tangent bundles  
Rui Albuquerque17:20-17:40 Jordan type of Artinian Gorenstein algebras and related invariants  
Pedro Macias Marques17:40-18:00 Shintani Descent and Supercharacter Theory  
João Dias18:00-18:20 Modelos descritos por equações diferenciais para o controlo da propagação da Malária em Angola  
Infeliz Carvalho Coxé **ONLINE****18:20-18:30 Closing of the Meeting**

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